Maulana Azad National Urdu University

Syllabus for Ph.D. **(Mathematics)** Entrance Test

Analysis

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum, Monotonic functions, continuity ,uniform continuity, differentiability, Mean Value Theorems, Riemann Integrals, sequences and series of functions. Metric Spaces-compactness, connectedness, Riemann Stieltjes Integrals, functions of bounded variation, Functions of several variables, Lebesgue measure and Lebesgue Integrals.
Normed linear Spaces, Banach Spaces-Hahn Banach Theorem, Principle of Uniform boundedness, Open Mapping and Closed graph theorem. Hilbert spaces- Basic definitions and properties of Inner product and Hilbert spaces. Orthogonal complements and projection theorem Reflexivity, Operators in Hilbert spaces.
Topology: Topological spaces-open bases – Finite Intersection property-compact spaces-separation axioms and connected spaces.

Algebra

Algebra: Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, congruences, Euler's Ø- function. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Field extensions, splitting fields and Galois Theory.

Linear Algebra

Vector Spaces – Definition and examples, Subspaces , related theorems and problems, Quotient Space, Linear dependent and independent set of vectors , Basis and Dimensions, Linear Transformation - Rank and Nullity of Linear Transformation, Matrix of a Linear Transformation. Eigen Values, Eigen Vectors, Characteristic polynomials, Minimal Polynomials of linear transformations, Cayley –Hamilton theorem, Results on Characteristic and minimal polynomial related to eigen values and eigen vectors. Inner Product Space - Unitary, Adjoint, self Adjoint, Normal Operators.

Mechanics

Moments of inertia, kinetic energy, angular momentum, mechanics of a particle and system of particles, kinematics of a rigid body, Euler's angles, Euler's dynamical equations, two dimensional motion of a rigid body, compound pendulum, constraints. D'Alembert's principle, Lagrange's equations of motion, techniques of calculus of variations. Hamilton's principles, Hamilton's equations of motion, contact transformation, Lagrange's and Poison brackets, integral in variances, Hamilton-Jacobi Poisson equations.

Ordinary Differential Equations

Existence & uniqueness theorem of solution of initial value problems for second and higher order differential equations. Series solution of second order linear differential equations near ordinary point, singularity and the solution in the neighbourhood of regular singular point, Euler equation and Frobenious method. Linear homogeneous boundary value problems, variation of parameters. Eigenvalues and Eigenfunctions, Sturm-Liouville boundary value problems.

Partial Differential Equations

Lagrange's and Charpit's general method for solving PDE's, Cauchy problem for first order PDE's, Classification of second order PDE's, general solution of higher order PDE's with constant coefficients, method of separation of variables for Laplace, heat and wave equations.

Complex Analysis

Representation of complex numbers, analytic function, Cauchy Riemann equations, power series. Some elementary functions, Harmonic functions. Cauchy's theorem, Cauchy's integral formula, Cauchy's inequality. Residues and poles, classification of isolated singularities, Taylor's and Laurent series. Cauchy Residue theorem. Application of residue theorem in evaluation of improper real integrals and evaluation of sum.

Numerical Analysis :

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and Spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations and Integral Equations:

Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations. Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Probability and Statistics:

Random experiments, Sample spaces, Sets, Events, Algebras. Elements of combinatorial analysis. Classical definition and calculation of Probability, Independence of events. Theorems on Probability- Conditional Probability, Multiplication theorem of Probability-Baye's theorem.

Random variables; Distribution functions, Moments, Probability and Moment generating functions. Sampling Theory – Types of sampling, Tests on small samples– The t- Distribution, Z- test for correlation, The F- distribution and Variance Ratio test. Chi Square Distribution- Limiting form of χ^2 - distribution. Chi Square test of independence, χ^2 test of goodness of fit.